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CURVED FLOW OF A VISCOUS LIQUID IN AN AXIRADIAL CHANNEL

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The motion of a viscous liquid is an axiradial channel with curving of the flow is investigated with the aid of the implicit method of splitting according to spatial variables. The effect of the Reynolds number and of the intensity of the curving on the formation of regions of reverse flows is established.

The nature of the flow of a liquid or gas in turbine channels and various kinds of vortex installations depends largely on the initial curving of the flow and the regularity of its change. It was established [1] that even when a model of ideal gas in axiradial channels is used, regions with reverse flow may be obtained. The disposition and intensity of these regions are determined by the flow parameters at the inlet.

Tret'yakov and Yagodkin [2] investigated viscous curved flow of liquid in annular channels. They showed that the intensity of the curvature of the flow has a substantial effect on the magnitude of the frictional surface tension and the formation of zones of reverse flows.

The present work is an investigation of curved flow of a liquid in axiradial channels on the basis of the Navier–Stokes equations.

Let us examine the steady motion of a viscous incompressible liquid in an axiradial channel whose meridional section is formed by some piecewise smooth curved lines AB, BC, CD, and AD (Fig. 1). The z axis is the axis of symmetry of the channel.

We use a system of cylindrical polar coordinates. We assume that in the plane of the channel section there exists some pole O (R_0, z_0) for which the following correlation between the cylindrical (R, φ, z) and cylindrical polar coordinates (r, φ, θ) of an arbitrary point M (Fig. 1) exists: $R = R_0 - r\cos\theta$, $\varphi=\varphi$, $z = z_0 + r\sin\theta$, where φ is the azimuth angle.

The boundaries AD, AB, BC, and DC of the meriodional channel section are specified by the equations $\theta = \theta_1(r)$, $r = r_1(\theta)$, $\theta = \theta_2(r)$, $r = r_2(\theta)$, respectively. All these functions have to be piecewise smooth.

We assume that the flow is axisymmetric, and then we write the dimensionless Navier–Stokes equations of the examined laminar motion of an incompressible liquid in a system of cylindrical polar coordinates in the form

$$\frac{u}{r}\frac{\partial u}{\partial\theta} + v\frac{\partial u}{\partial r} + \frac{uv}{r} - \frac{w^2}{R}\sin\theta = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \frac{1}{\mathrm{Re}}\left[\Delta u - \left(\frac{1}{r^2} + \frac{\sin^2\theta}{R^2}\right)u + \frac{2}{r^2}\frac{\partial v}{\partial\theta} + \left(\frac{1}{r} + \frac{\cos\theta}{R}\right)\frac{\sin\theta}{R}u\right], \quad (1)$$

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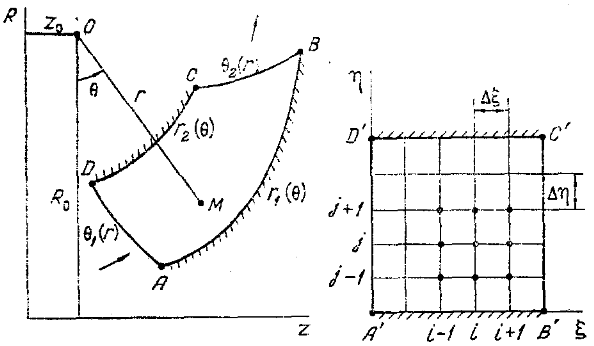

Fig. 1. Meridional section of an axiradial channel in the plane of the coordinates R, z and ζ, η.

$$\frac{u}{r}\frac{\partial v}{\partial \theta}+v\frac{\partial v}{\partial r}-\frac{u^2}{r}+\frac{w^2}{R}\cos\theta=-\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left[\Delta v-\left(\frac{1}{r^2}+\frac{\cos^2\theta}{R^2}\right)v-\frac{2}{r^2}\frac{\partial u}{\partial \theta}-\left(\frac{1}{r}-\frac{\cos\theta}{R}\right)\frac{\sin\theta}{R}u\right],$$

$$\frac{u}{r}\frac{\partial w}{\partial \theta}+v\frac{\partial w}{\partial r}+(u\sin\theta-v\cos\theta)\frac{w}{R}=\frac{1}{\mathrm{Re}}\left[\Delta w-\frac{w}{R^2}\right], \tag{3}$$

$$\frac{\partial (Ru)}{\partial \theta}+\frac{\partial (Rrv)}{\partial r}=0. \tag{4}$$

Here Δ is the operator,

$$\Delta\equiv\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{rR}\frac{\partial}{\partial \theta}+\left(\frac{1}{r}-\frac{\cos\theta}{R}\right)\frac{\partial}{\partial r}.$$

The system of equations (1)-(4) is solved with the following boundary conditions:

$$u=v=w=0 \text{ on the walls } AB \text{ and } DC;$$
$$u=U(r),\quad v=V(r),\quad w=W(r) \text{ at the inlet } AD. \tag{5}$$

On the outlet edge of the channel BC "soft" conditions are specified. We will seek the solution of the problem in the variables: flow function ψ — vorticity ω — azimuthal speed w. Then

$$u=-\frac{1}{R}\frac{\partial \psi}{\partial r},\quad v=\frac{1}{rR}\frac{\partial \psi}{\partial \theta},\quad \omega=\frac{1}{r}\left[\frac{\partial v}{\partial \theta}-\frac{\partial (ru)}{\partial r}\right].$$

Equation (4) is satisfied identically, and (1)-(3) are changed to the form

$$\frac{\partial}{\partial \theta}\left(\frac{1}{rR}\frac{\partial \psi}{\partial \theta}\right)+\frac{\partial}{\partial r}\left(\frac{r}{R}\frac{\partial \psi}{\partial r}\right)=r\omega, \tag{6}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\omega}{R}\frac{\partial \psi}{\partial \theta}\right)-\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\omega}{R}\frac{\partial \psi}{\partial r}\right)+\frac{1}{R}\left(\frac{\cos\theta}{r}\frac{\partial w^2}{\partial \theta}+\sin\theta\frac{\partial w^2}{\partial r}\right)=\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\frac{r}{R}\frac{\partial(\omega R)}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial \theta}\frac{1}{R}\frac{\partial(\omega R)}{\partial \theta}\right], \tag{7}$$

$$\frac{1}{rR^2}\frac{\partial}{\partial r}\left(Rw\frac{\partial \psi}{\partial \theta}\right)-\frac{1}{rR^2}\frac{\partial}{\partial \theta}\left(Rw\frac{\partial \psi}{\partial r}\right)=\frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\frac{r}{R}\frac{\partial(wR)}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial \theta}\frac{1}{R}\frac{\partial(wR)}{\partial \theta}\right]. \tag{8}$$

We transform the flow region ABCD with arbitrary curved boundaries into the square A'B'C'D' (Fig. 1) with a side of unit length. For that we carry out the following transformation with nonzero Jacobian:

$$\xi=\frac{\theta-\theta_1(r)}{\theta_2(r)-\theta_1(r)},\quad \eta=\frac{r-r_1(\theta)}{r_2(\theta)-r_1(\theta)}. \tag{9}$$

We assume that the functions $r_1(\theta)$, $r_2(\theta)$, $\theta_1(r)$, and $\theta_2(r)$ are stipulated analytically or in tabulated form for the intervals of changes of the arguments θ and r that are necessary for (9).

In a corresponding way Eqs. (6)-(8) are transformed in the new variables ξ, η, and we write the boundary conditions (5) as follows:

$$\begin{gathered}
\psi = -\eta_r^{-1}\int_0^{\eta} U(\eta)\,R d\eta,\ \omega = -\eta_r \frac{dU}{d\eta},\ w = W(\eta) \quad \text{for} \quad \xi = 0,\\
\psi = 0,\ \omega = \frac{1}{R}\left(\eta_r^2 + \frac{\eta_\theta^2}{r^2}\right)\frac{\partial^2\psi}{\partial\eta^2},\ w = 0 \quad \text{for} \quad \eta = 0,\\
\psi = \psi^* \equiv -\eta_r^{-1}\int_0^{1} U(\eta)\,R d\eta,\ \omega = \frac{1}{R}\left(\eta_r^2 + \frac{\eta_\theta^2}{r^2}\right)\frac{\partial^2\psi}{\partial\eta^2},\ w = 0 \quad \text{for} \quad \eta = 1,\\
\frac{\partial\psi}{\partial\xi} = \frac{\partial\omega}{\partial\xi} = \frac{\partial w}{\partial\xi} = 0 \quad \text{for} \quad \xi = 1.
\end{gathered} \tag{10}$$

Here, $\eta_r \equiv \partial\eta/\partial r$, $\eta_\theta \equiv \partial\eta/\partial\theta$. In the boundary conditions (10) it was accepted that $\theta_1(r) \equiv 0$, $V(r) \equiv 0$.

To obtain the finite-difference analog of the Navier–Stokes equations, we construct a the grid of coordinate lines ξ_i = const, η_j = const with constant steps $\Delta\xi$, $\Delta\eta$: $\xi_i = (i - 1)\Delta\xi$, $\eta_j = (j - 1)\Delta\eta$; $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, M$.

We find the steady-state solution of the problem by the method of establishment. It is assumed that the sought magnitudes depend on some time parameter t. The limit value of these magnitudes for $t \to \infty$ and boundary conditions not dependent on time is taken as the sought steady-state solution of the problem under examination.

We introduce the vector $f = \{wR, \psi, \omega R\}$ into the examination. We denote its value at the point $(\xi_i, \eta_j)^n$, where n is the number of the time-dependent layer, by $f_{i,j}^n$. The convective derivatives in the Navier–Stokes equations at the point $(\xi_i, \eta_j)^n$ will be approximated by the scheme of the "donor cell" [3] that takes the direction of the flow into account, and the second and mixed derivatives will be approximated on the nine-point pattern (Fig. 1). As a result we obtain the finite-difference schema in the form

$$H(f_{i,j}^n) = 0, \quad H \equiv \begin{Vmatrix} H_1 \\ H_2 \\ H_3 \end{Vmatrix}, \tag{11}$$

$i = 1, 2, \ldots, N - 1$; $j = 2, \ldots, M - 1$, correlating the values of the sought lattice functions on the n-th time-dependent layer.

For the transition from the n-th time-dependent layer to the (n + 1) layer we introduce the correction ΔX and ΔY to the vector of the sought functions f^n (we omit the subscripts), so that

$$f^{n+\frac{1}{2}} = f^n + \Delta X; \quad f^{n+1} = f^{n+\frac{1}{2}} + \Delta Y.$$

The equations for the corrections are determined by the initial differential equations written in nonsteady form and split into the directions ξ and η:

$$A_x \frac{2\Delta X}{\Delta t} + B_x \frac{\partial \Delta X}{\partial \xi} + C_x \frac{\partial^2 \Delta X}{\partial \xi^2} + H(f^n) = 0, \tag{12}$$

$$A_y \frac{2\Delta Y}{\Delta t} + B_y \frac{\partial \Delta Y}{\partial \eta} + C_y \frac{\partial^2 \Delta Y}{\partial \eta^2} + H(f^{n+\frac{1}{2}}) = 0, \tag{13}$$

where

$$A_x = A_y = A; \quad B_x = \frac{u\xi_\theta}{r} B; \quad B_y = \left(\frac{u\eta_\theta}{r} + v\eta_r\right) B; \quad C_x = \left(\frac{\xi_\theta}{r}\right)^2 C;$$

$$C_y = \left(\frac{\eta_\theta^2}{r^2} + \eta_r^2\right) C;$$

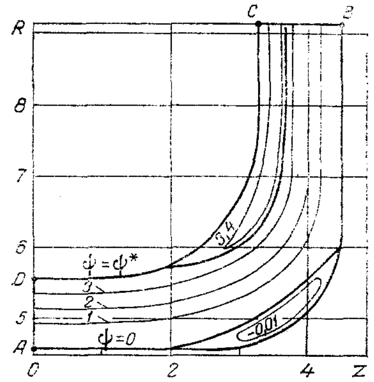

Fig. 2

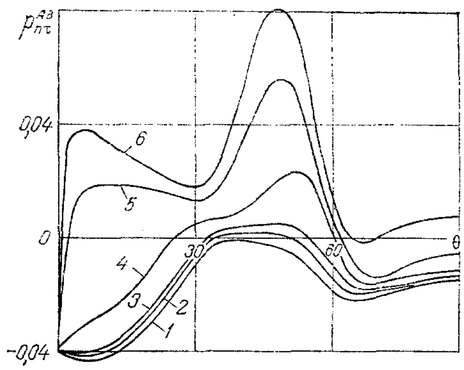

Fig. 3

Fig. 2. Flow lines in an axiradial channel with Re = 2000 and no curving of the flow.

Fig. 3. Change of the longitudinal component of the tangential stress on the inner channel wall and with Re = 100 and different curvings: 1) $K_0 = 0$ ($W \equiv 0$); 2) 0.85 ($W = 1 - 0.3\eta$); 3) 1.15 ($W = 1 + 0.3\eta$); 4) 2.3 ($W = 2 + 0.6\eta$); 5) 4.25 ($W = 5 - 1.5\eta$); 6) 5.75 ($W = 5 + 1.5\eta$). θ, deg.

$$A = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{Vmatrix}; \quad B = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{Vmatrix}; \quad C = \begin{Vmatrix} -\frac{1}{\mathrm{Re}} & 0 & 0 \\ 0 & \frac{r}{R} & 0 \\ 0 & 0 & -\frac{1}{\mathrm{Re}} \end{Vmatrix};$$

a is a parameter that changes the time scale in the equations for the component ψ; $\xi_\theta \equiv \partial\xi/\partial\theta$.

The boundary conditions for the vectors ΔX and ΔY were formulated with a view to (10), and the expression for the correction for vorticity on the wall was written with second order of accuracy, and a special correction of the longitudinal velocity was carried out at the points of the difference lattice that are one step distant from the wall [4]. In the calculation of flow with complex configuration and with regions of reverse flow such a way of stating the boundary conditions proved to be more effective.

The solution of Eqs. (12), (13) was effected by the implicit difference schema by two successive vector matchings in the directions of the ξ and η axes. As a result we determined the value of the vector f^{n+1} on the new time-dependent layer. The process of establishment was considered completed when the change of the sought functions became smaller than a previously stipulated small number ε.

The calculation method described above was used for investigating the motion of a liquid in an axiradial channel with a turn of the flow through 90°. The coordinates of the boundary points of the channel illustrated in Fig. 2 were stipulated in tabular form with successive smoothing with the aid of spline functions [5]. At the channel inlet we adopted with profile of the longitudinal velocity $U(\eta)$ corresponding to flow in a cylindrical annular pipe, and of the circumferential velocity according to the linear regularity $W(\eta) = \alpha + \beta\eta$, where α, β are constants.

Thus, when the geometry of the channel is known, the principal parameters of the problem are the Reynolds number and the values of α and β.

The calculations of the flow were carried out in the range of the so-called "moderate" Reynolds numbers $10 \leqslant \mathrm{Re} \leqslant 2000$ where the motion of the liquid is of laminar nature. The Reynolds number was determined according to the width of the inlet section of the channel and the maximum longitudinal velocity in this section.

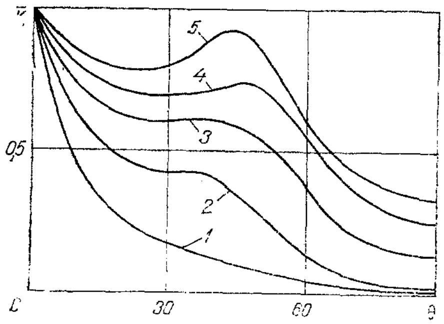

Fig. 4. Change of the relative curving of the flow in the radial sections of the channel for $W(\eta) = 2 + 0.6\eta$: 1) Re = 10; 2) 25; 3) 50; 4) 100; 5) 200.

First of all we will describe the results of the calculation of the flow in an axiradial channel when there is no curving of the flow at the inlet. These calculations showed that for Re < 100 the flow in the channel is of unidirectional nature without zones of reverse motions. When Re ≈ 100, a closed zone of reverse motion of the liquid appears on the inner wall AB of the channel in the region of its turn, and this zone increases with increasing Re. When the Reynolds number Re ≈ 200, the closed region of reverse flow also appears on the outer wall DC when $\theta \approx 45°$. With increasing Reynolds number this region spreads downstream fairly intensely, and when Re ≈ 500, we find that the liquid flows in through the part of the outlet section adjacent to point C. Figure 2 shows the regions of developed reverse flow corresponding to Re = 2000.

If the flow at the channel inlet is curved, then the above-described development of the zones of reverse flow becomes more pronounced with increasing parameter K_0 characterizing the intensity of the curving of the flow at the initial section:

$$K_0 = \int_{r_D}^{r_A} W(r)\, dr.$$

Thus, with $K_0 = 1.15$ ($\alpha = 1$; $\beta = 0.3$) the flow pattern analogous to the one in Fig. 2 occurs already with Re = 500. With values $K_0 = 4$-5 and Re ≈ 100, the regions of reverse flow approaches the inlet section of the channel. This is clearly visible in Fig. 3 which shows the distribution of the dimensionless longitudinal component of the tangential stress $p_{n\tau}$ on the lower channel wall AB (τ, n are the tangent unit vector and the unit vector of the normal to the wall, respectively). If we assume that at the points of detachment and attachment of the flow $p_{n\tau} = 0$, it follows from Fig. 3 that with strong initial curving of the flow ($K_0 = 5.75$) reverse flow occupies practically the entire region near the wall AB.

Under real conditions this means that liquid behind the outlet section of the channel may penetrate through the zone of reverse flow into elements of the installation situated ahead of the inlet to the channel. All these phenomena are undesirable, they impair the efficiency of operation of the channel.

Figure 4 shows the dependence of $\overline{K} = K_0^{-1}\int_{r_2}^{r_1} w\, dr$ on the angle θ in the radial sections (θ = const) of the channel for the curving $W(\eta) = 2 + 0.6\eta$ ($K_0 = 2.3$). With Re = 10 this dependence is of a monotonically decreasing nature owing to the substantial dissipative effect of the viscosity forces. With increasing Re this dependence becomes ever more nonmonotonic because the effect of the viscosity forces becomes weaker, and the radial section of the channel in some parts then lies in the region of higher values of curving. Nearer the outlet part of the channel the dependence $\overline{K}(\theta)$ is of a monotonically decreasing nature, also because of the damping effect of viscosity.

The suggested method may be used in the process of designing channels. In the course of a numerical experiment it is possible by deliberately changing channel geometry to attain that there are no regions of reverse flow, and thereby to improve the efficiency of the channel.

In conclusion we want to point out that the calculations were carried out with a lattice 21 × 21. Control calculations with a lattice 41 × 41 showed that the relative deviation of the sought values in the entire range of the calculation lay within 3%.

NOTATION

R, φ, z, cylindrical coordinates; r, φ, θ, cylindrical polar coordinates; R_0, z_0, coordinates of the point 0; v, w, u, velocity components in the system of cylindrical polar coordinates; ψ, flow function; ω, vorticity; ξ, η, new variables; p, pressure; Re, Reynolds number; $\Delta\xi$, $\Delta\eta$, steps of the lattice; H, vector of discrepancies of the Navier–Stokes equations; ΔX, ΔY, vectors of corrections for the sought solution.

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FINITE-DIFFERENCE SOLUTION OF THE CONJUGATE HEAT TRANSFER, NATURAL CONVECTION, AND SOLIDIFICATION PROBLEM

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A mathematical model of the solidification of a binary melt under convection conditions with a two-phase zone taken into account is formulated on the basis of average transfer equations. Results of a numerical solution are presented.

The solidification of binary alloys is characterized by the presence of an intermediate domain which is a heterogeneous mixture of the liquid and solid phases and a so-called two-phase zone. As is known, the reason for the formation of such a zone [1] is the development of concentration and kinetic supercooling.

Two approaches exist for the compilation of the heat, mass, and momentum balance equations for the two-phase zone in a mathematical description of melt solidification. In one formal replacement of the heterogeneous by a homogeneous medium is assumed, where the thermophysical parameters of the latter are defined as average. Then the process is described by equations for a homogeneous medium. Here can also be referred the method of using effective physical, experimentally determined, parameters in these equations.

The second approach, which possesses great generality, is based on the mechanics of multiphase media [2] and assumes the examination of the two-phase zone in the approximation of average macrocontinuums [3]. The selection of the averaging method is also quite important here. Averaging over the volume of macropoints, executed according to known rules, is the most natural.

In this paper, both approaches are applied in formulating the problem. The average heat and mass transfer differential equations

$$\rho\,[\xi c_1 + (1-\xi)\,c_2]\,\frac{\partial T}{\partial t} + c_2\rho\,(1-\xi)\,\vec{V}\nabla T = \nabla\,[\lambda_1\nabla\xi T + \lambda_2\nabla\,(1-\xi)\,T] + L\rho\,\frac{\partial\xi}{\partial t}, \tag{1}$$

$$(1-\xi)\,\frac{\partial C}{\partial t} = \nabla\,[D\nabla\,(1-\xi)\,C] + (1-k)\,C\,\frac{\partial\xi}{\partial t} \tag{2}$$

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